COUNTERPOSITION AND NEGATIVE REFRACTION DUE TO UNIFORM MOTION

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ABSTRACT: Refraction of obliquely incident plane waves due to the interface of a vacuous half–space and a half–space occupied by a simply moving, nondissipative, isotropic dielectric–magnetic medium is considered, when the medium's velocity lies parallel to the interface and in the plane of incidence. Counterposition of the refracted wavevector and time–averaged Poynting vector occurs when the medium's velocity is sufficiently large in magnitude and makes an obtuse angle with the incident wavevector. The counterposition induced by relative motion occurs whether the refraction is negative or positive when the medium is at rest.

1. INTRODUCTION

When a plane wave is incident upon the planar interface of two homogeneous mediums, the refracted wavevector and time–averaged Poynting vector can emerge on opposite sides of the normal to the interface. This phenomenon is called *counterposition*. Conditions for the

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occurrence of counterposition in uniaxial dielectric–magnetic mediums have been established [1]. Under the title of amphoteric refraction, counterposition has been confused with negative refraction [2]. Refraction — whether negative or positive — concerns the orientation of the refracted wavevector relative to the normal to the interface, as per a law often attributed to Willebrord van Snel van Royen [3]. The orientation of the refracted time–averaged Poynting vector is irrelevant to whether or not the refraction is negative or positive.

In this Letter, we establish the prospects for counterposition due to the interface of a vacuous half–space and a half–space occupied by a simply moving, nondissipative, isotropic dielectric–magnetic medium, when the medium's velocity lies parallel to the interface and in the plane of incidence. In particular, we show that counterposition and negative refraction are indeed distinct. For background details of planewave propagation in moving mediums, the reader is referred to standard works [3, 4, 5].

2. ANALYSIS

Suppose that a plane wave is launched with wavevector $\boldsymbol{k_i} = k_i \hat{\boldsymbol{k_i}}$ from vacuum towards a half–space occupied by an isotropic, nondissipative, dielectric–magnetic medium. Let this medium move at constant velocity $\boldsymbol{v} = v\hat{\boldsymbol{v}}$, parallel to the interface and in the plane of incidence. With respect to an inertial frame of reference that moves with the same velocity $\boldsymbol{v}$ with respect to the laboratory frame of reference wherein $\boldsymbol{k_i}$ is specified, the refracting medium is characterized by relative permittivity ϵ_r and relative permeability μ_r. The condition $\epsilon_r \mu_r \geq 1$ is assumed in order to exclude the possibility of evanescent plane waves.

The angle ϕ_t between the refracted wavevector $\boldsymbol{k_t} = k_t \hat{\boldsymbol{k_t}}$, as observed from the laboratory frame of reference, and the unit vector $\hat{\boldsymbol{q}}$ normal to the interface is related to the angle of incidence

$$\phi_i = \cos^{-1}\left(\hat{\boldsymbol{k}}_i \cdot \hat{\boldsymbol{q}}\right) \tag{1}$$

by [3]

$$\phi_t = \sin^{-1}\left(\frac{k_0 \sin \phi_i}{k_t}\right), \tag{2}$$

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where

$$k_t = \begin{cases} k_0 \left\{ 1 + \xi \left[1 - \beta \left(\hat{\boldsymbol{k}}_{\boldsymbol{i}} \cdot \hat{\boldsymbol{v}} \right) \right]^2 \right\}^{1/2} & \text{for} \quad \epsilon_r, \mu_r > 0 \\[2em] -k_0 \left\{ 1 + \xi \left[1 - \beta \left(\hat{\boldsymbol{k}}_{\boldsymbol{i}} \cdot \hat{\boldsymbol{v}} \right) \right]^2 \right\}^{1/2} & \text{for} \quad \epsilon_r, \mu_r < 0 \end{cases} \tag{3}$$

is the wavenumber of the refracted wave, k_0 is the wavenumber in vacuum,

$$\xi = \frac{\epsilon_r \mu_r - 1}{1 - \beta^2}, \tag{4}$$

and $\beta = v\sqrt{\epsilon_0 \mu_0}$ with ϵ_0 and μ_0 being the permittivity and permeability of vacuum.

Let us consider case where ϵ_r and μ_r are positive–valued. Then, $0 < \phi_t < \pi/2$ for all $\phi_i \in (0, \pi/2)$. That is, the refraction is always positive $\forall \beta \in (-1, 1)$, as is illustrated schematically in Fig. 1. Plots of ϕ_t against $\beta \in (-1, 1)$ for three values of ϕ_i are provided in Fig. 2.

The time–averaged Poynting vector of the refracted plane wave is given by [3]

$$\boldsymbol{P}_t = P_t \hat{\boldsymbol{P}}_t = \left(\frac{1}{\mu_r} |C_1|^2 + \epsilon_r |C_2|^2 \right) (\boldsymbol{k_t} \times \hat{\boldsymbol{v}})^2 \left[\boldsymbol{k_t} + \xi\beta \left(k_0 - \beta \boldsymbol{k_t} \cdot \hat{\boldsymbol{v}} \right) \hat{\boldsymbol{v}} \right], \tag{5}$$

where C_1 and C_2 are constants. The angle

$$\phi_P = \tan^{-1} \left(\frac{\hat{\boldsymbol{P}}_t \cdot \boldsymbol{v}}{|v| \hat{\boldsymbol{P}}_t \cdot \hat{\boldsymbol{q}}} \right) \tag{6}$$

between $\hat{\boldsymbol{q}}$ and $\hat{\boldsymbol{P}}_t$ is plotted in Fig. 3 against $\beta \in (-1, 1)$ for three values of ϕ_i. The orientation of the refracted time–averaged Poynting vector rotates towards the direction of motion as β increases from -1. Clearly, counterposition arises $\forall \beta < \tilde{\beta}$ where $\tilde{\beta}$ is some negative number that depends, among other quantities, on ϕ_i.

Now let us turn to the scenario wherein $\epsilon_r < 0$ and $\mu_r < 0$, as schematically represented in Fig. 4. As a consequence of both ϵ_r and μ_r being negative–valued, we have $-\pi/2 < \phi_t < 0$ for all $\phi_i \in (0, \pi/2)$. That is, the refraction is always negative $\forall \beta \in (-1, 1)$. The plots in Fig. 5 of ϕ_t against $\beta \in (-1, 1)$ for three values of ϕ_i illustrate that conclusion.

The corresponding orientation angles for the refracted time–averaged Poyting vector are graphed against $\beta \in (-1, 1)$ in Fig. 6. As is the case for the positively refracting scenario, we see that counterposition arises $\forall \beta < \tilde{\beta}$ where $\tilde{\beta}$ is negative. However, in contrast to the

positively refracting scenario, the refracted time–averaged Poynting vector rotates against the direction of motion as β increases.

3. CONCLUDING REMARKS

Thus, counterposition may be induced in an isotropic dielectric–magnetic medium by relative motion at constant velocity — whether the medium is positively or negatively refracting when at rest. Thereby, the distinction between counterposition and negative refraction is further emphasized [2, 6]. Also, in the scenario considered here, we note that the phase velocity of the refracted plane wave is positive when positive refraction occurs and negative when negative refraction occurs. Hence, negative phase velocity is not induced by relative motion parallel to the medium interface [7]. In the absence of relative motion (i.e., $\beta = 0$) the refracted wavevector and time–averaged Poynting vector are parallel for $\epsilon_r, \mu_r > 0$ and anti–parallel for $\epsilon_r, \mu_r < 0$, and accordingly counterposition cannot occur, as is confirmed by Figs. 2, 3, 5 and 6.

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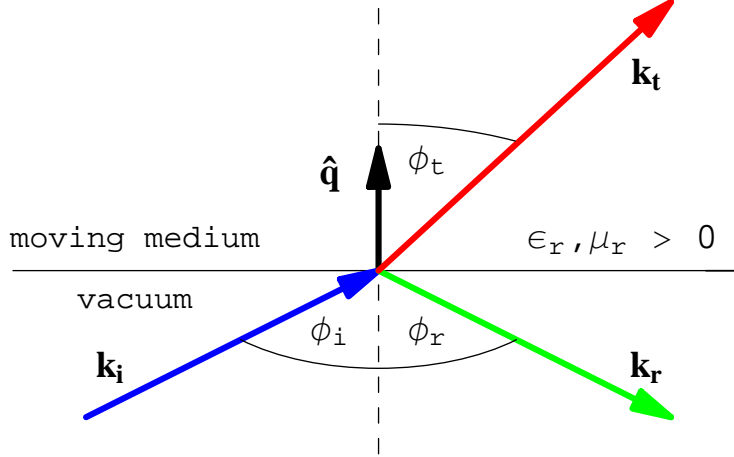
Figure 1: A plane wave with wavevector k_i is incident from vacuum onto a half–space occupied by a simply moving medium at an angle ϕ_i with respect to the unit vector $\hat{q}$ normal to the planar interface. The moving medium is characterized by relative permittivity $\epsilon_r > 0$ and relative permeability $\mu_r > 0$ in a comoving frame of reference. As observed in the non–comoving (laboratory) frame of reference wherein the incident plane wave is specified, the refracted wavevector k_t makes an angle ϕ_t with $\hat{q}$.
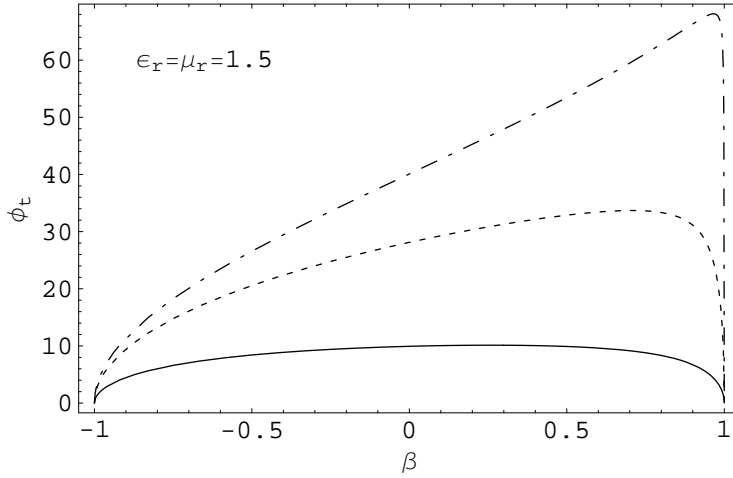
Figure 2: The angle of refraction ϕ_t (in degree) plotted as a function of $\beta \in (-1, 1)$, when the angle of incidence $\phi_i = 15°$ (solid curves), $45°$ (dashed curves) and $75°$ (broken dashed curves); $\epsilon_r = \mu_r = 1.5$.
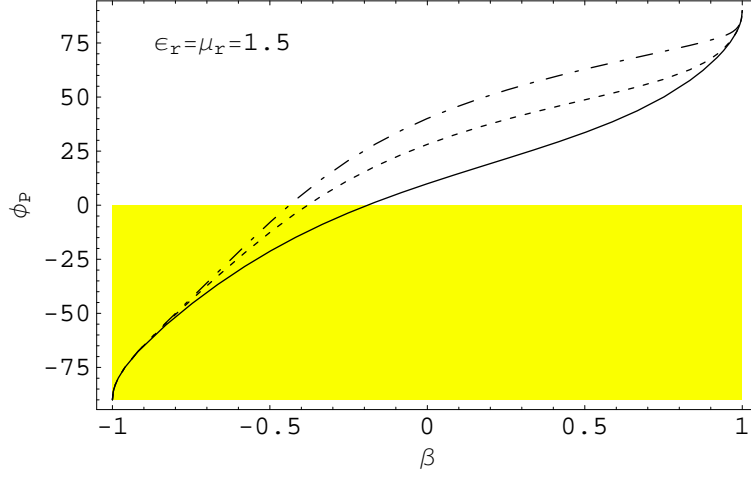
Figure 3: As Figure 2, but for the angle ϕ_P (in degree) between the refracted time–averaged Poynting vector $\hat{\boldsymbol{P}}_t$ and the unit vector $\hat{\boldsymbol{q}}$. The counterposition regime $\phi_P < 0°$ is shaded.
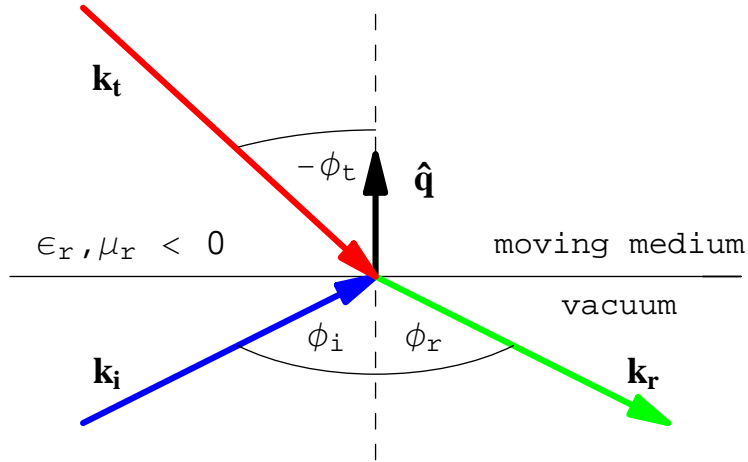

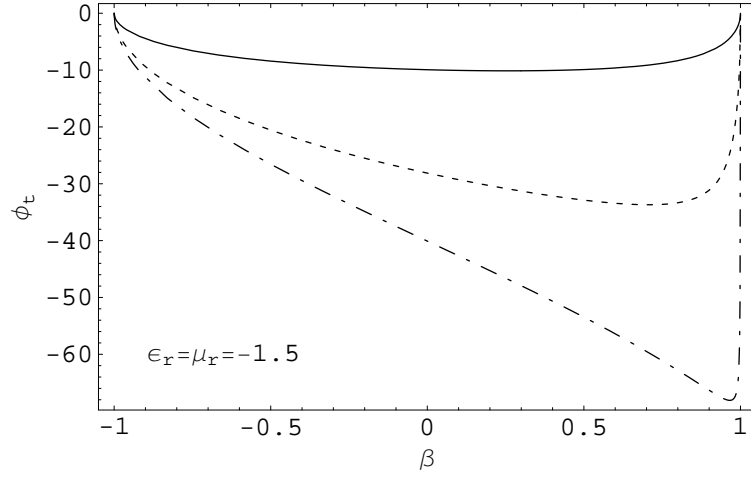Figure 4: As Figure 1 but for $\epsilon_r < 0$ and $\mu_r < 0$.

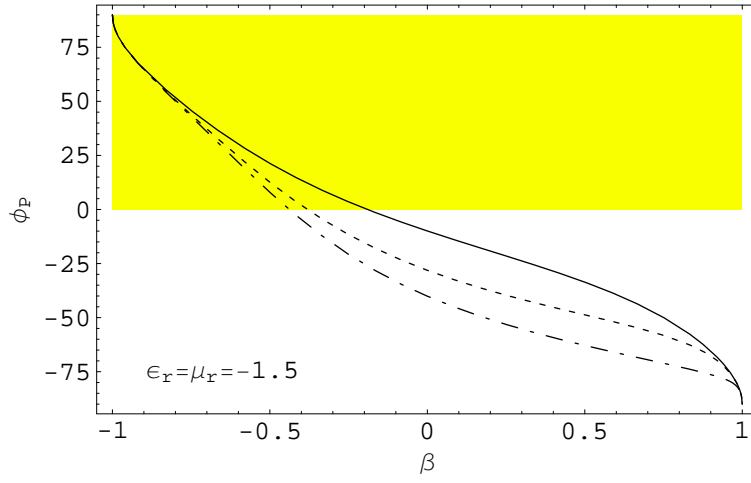Figure 5: As Figure 2 but for $\epsilon_r = \mu_r = -1.5$.

Figure 6: As Figure 3 but for $\epsilon_r = \mu_r = -1.5$. The counterposition regime $\phi_P > 0°$ is shaded.